Efficient Evaluation of Lazy Programs, or Compilation of Call-by-Need

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Declaimer

This presentation is based on

Simon P. Jones and David R. Lester's
Implementing Functional Languages: a Tutorial

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Implementing Functional Languages: a Tutorial

For a "modernized" version of its appendix,
one can see https://github.com/Ailrun/core-lang-haskell

Advantages of Lazy Programs

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- Bring us efficient persistent data structures

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- Make it easier to deal with recursion when using combinator libraries
- Bring us efficient persistent data structures
- Provide a way to encode co-data (e.g. streams)

The Question

Are these advantages real?

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Is it possible to efficiently execute a lazy program?

Difficulties of Efficient Evaluation of Lazy Programs

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1. Sharing

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2. (Redirection)

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3. **Instantiation**

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- What are These Difficulties?

Difficulties of Efficient Evaluation of Lazy Programs

1. Sharing
2. (Redirection)
3. **Instantiation**

- What are These Difficulties?
- How to Solve These Difficulties?

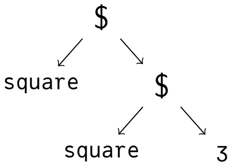
The First Difficulty: Sharing - 1

```
square x = x * x
main = square (square 3)
```

The First Difficulty: Sharing - 1

```
square x = x * x
main = square (square 3)
```

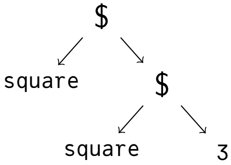
AST for `main`



The First Difficulty: Sharing - 1

```
square x = x * x
main = square (square 3)
```

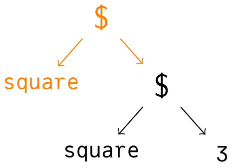
AST for `main`



How can we reduce this tree into `81`?

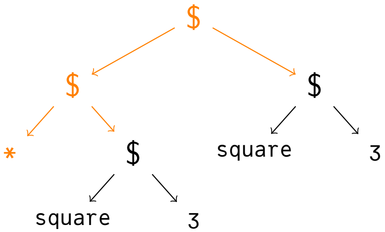
The First Difficulty: Sharing - 2

```
square x = x * x
main = square (square 3)
```



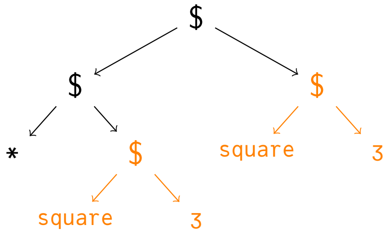
The First Difficulty: Sharing - 2

```
square x = x * x
main = square (square 3)
```



The First Difficulty: Sharing - 2

```
square x = x * x
main = square (square 3)
```

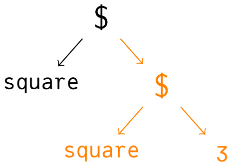


We wastefully repeat the computation!

The First Difficulty: Sharing - 3

Let's share `x` part

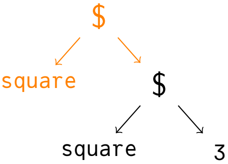
```
square x = x * x
main = square (square 3)
```



The First Difficulty: Sharing - 3

Let's share `x` part

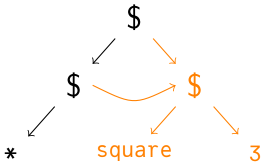
```
square x = x * x
main = square (square 3)
```



The First Difficulty: Sharing - 3

Let's share `x` part

```
square x = x * x
main = square (square 3)
```



The First Difficulty: Sharing - 3

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square x = x * x
main = square (square 3)
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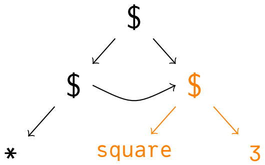

The First Difficulty: Sharing - 3

Let's share `x` part

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square x = x * x
main = square (square 3)
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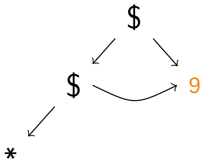


The Solution for Sharing

We reduces *a (directed) graph*, *not a tree*.

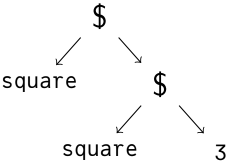
Moreover, we need to *update* a node
(so that multiple refereces share the evaluation cost)

Efficiency Requirement

Each graph reduction step should be as *local and small* as possible
(for further optimizations, machine-compilability, etc.)

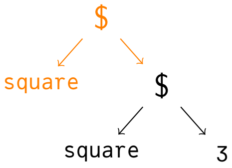
The Second Difficulty: Redirection - 1

```
id x = x
square x = (id x) * x
main = square (square 3)
```



The Second Difficulty: Redirection - 1

```
id x = x
square x = (id x) * x
main = square (square 3)
```



The Second Difficulty: Redirection - 1

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id x = x
square x = (id x) * x
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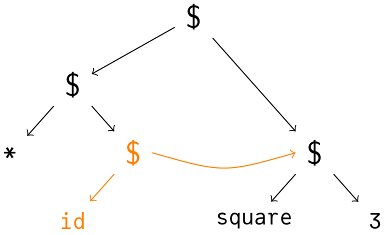

The Second Difficulty: Redirection - 1

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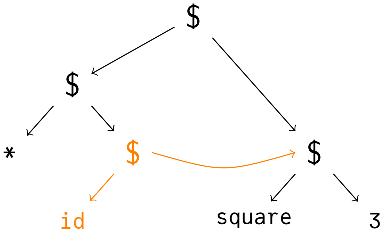
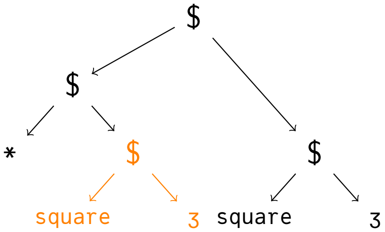
id x = x
square x = (id x) * x
main = square (square 3)
```



How should we reduce the application node for `id`?

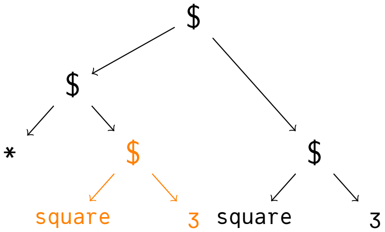
The Second Difficulty: Redirection - 1

```
id x = x
square x = (id x) * x
main = square (square 3)
```



The Second Difficulty: Redirection - 1

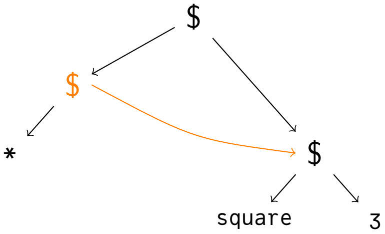
```
id x = x
square x = (id x) * x
main = square (square 3)
```



We lose sharing!

The Second Difficulty: Redirection - 1

```
id x = x
square x = (id x) * x
main = square (square 3)
```



The Second Difficulty: Redirection - 1

```
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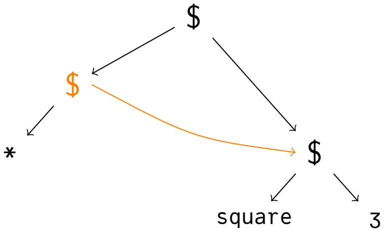
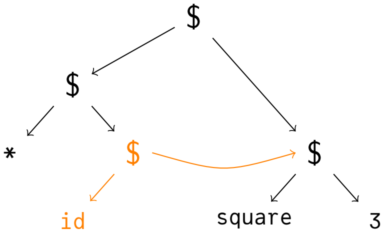
square x = (id x) * x
main = square (square 3)
```



We need to modify an ancestor (depending on the depth of `id`)

The Second Difficulty: Redirection - 1

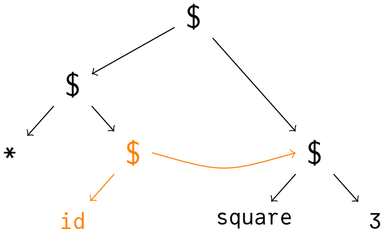
```
id x = x
square x = (id x) * x
main = square (square 3)
```



How should we handle this?

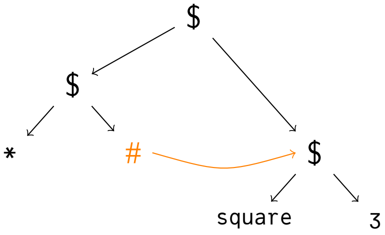
The Second Difficulty: Redirection - 2

```
id x = x
square x = (id x) * x
main = square (square 3)
```



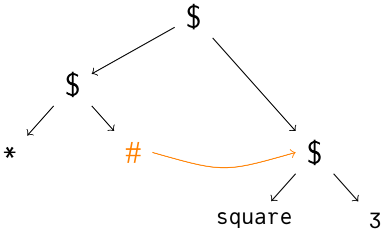
The Second Difficulty: Redirection - 2

```
id x = x
square x = (id x) * x
main = square (square 3)
```



The Second Difficulty: Redirection - 2

```
id x = x
square x = (id x) * x
main = square (square 3)
```



We follow this *redirection* node (#) when we reduce the parent

The Solution for Redirection

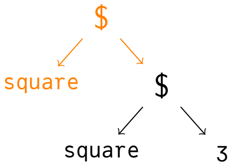
We introduce an “run-time only” node (#) to handle it

Efficiency Requirement - Again

Each graph reduction step should be as *local and small* as possible

Unfortunate "Change-all" Step

```
square x = x * x
main = square (square 3)
```



Unfortunate "Change-all" Step

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```
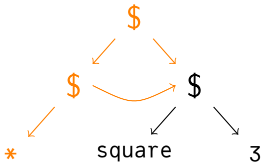
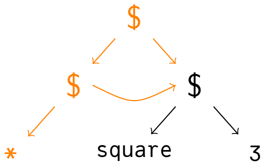



Unfortunate "Change-all" Step

```
square x = x * x
main = square (square 3)
```



This changes almost entire structure of graph!

Unfortunate "Change-all" Step - Specifically

When we instantiate a function definition as a part of a graph,
we need to analyze the current graph
and
to construct a new graph

The Main Difficulty: Instantiation

How can we divide this huge step into smaller steps?

The First Solution for Instantiation: G-Machine - 1

Let's start with construction of graph of `main` itself

```
square x = x * x
main = square (square 3)
```

The First Solution for Instantiation: G-Machine - 1

We construct it in a argument-first way, so start at `3`

```
square x = x * x
main = square (square 3)
```

3

The First Solution for Instantiation: G-Machine - 1

and then `square`

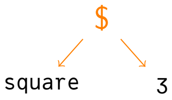
```
square x = x * x
main = square (square 3)
```

square 3

The First Solution for Instantiation: G-Machine - 1

Now we can form an application node

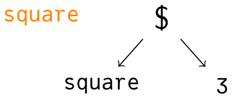
```
square x = x * x
main = square (square 3)
```



The First Solution for Instantiation: G-Machine - 1

Once we add another `square`

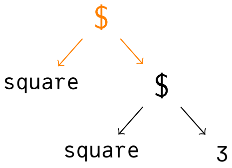
```
square x = x * x
main = square (square 3)
```



The First Solution for Instantiation: G-Machine - 1

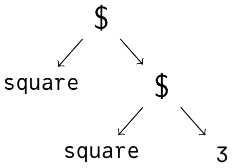
We can finish the main graph by constructing an application node

```
square x = x * x
main = square (square 3)
```



The First Solution for Instantiation: G-Machine - 2

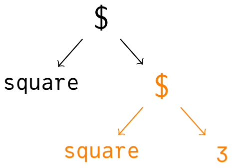
Now let's apply `square`



```
square x = x * x
main = square (square 3)
```

The First Solution for Instantiation: G-Machine - 2

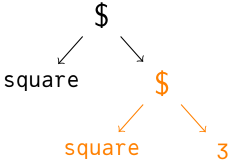
We construct the arugment first



```
square x = x * x
main = square (square 3)
```

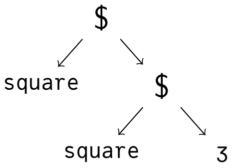
The First Solution for Instantiation: G-Machine - 2

To construct its application node, we need to construct the function node



```
square x = x * x
main = square (square 3)
```

The First Solution for Instantiation: G-Machine - 2

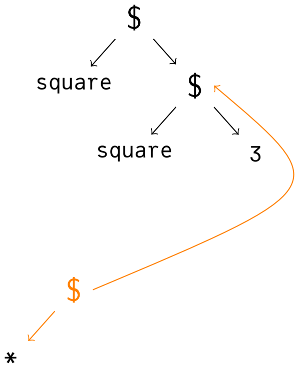


```
square x = x * x
main = square (square 3)
```

*

The First Solution for Instantiation: G-Machine - 2

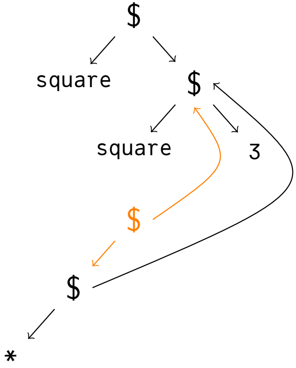
```
square x = x * x
main = square (square 3)
```



The First Solution for Instantiation: G-Machine - 2

and then the top-level application node

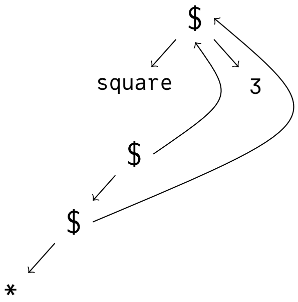
```
square x = x * x
main = square (square 3)
```



The First Solution for Instantiation: G-Machine - 2

Now, clean up the old root

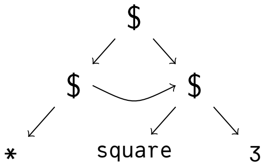
```
square x = x * x
main = square (square 3)
```



The First Solution for Instantiation: G-Machine - 2

After visual rearrange, it is clear that we get the expected graph

```
square x = x * x
main = square (square 3)
```



The First Solution for Instantiation: G-Machine - 3

Can we translate this into recordable code pieces?
Then we can “compile” `main` and `square` into those.

```
square x = x * x
main = square (square 3)
```

The First Solution for Instantiation: G-Machine - 3

Let's repeat the `main` construction first.

```
square x = x * x
main = square (square 3)
```

The First Solution for Instantiation: G-Machine - 3

We construct it in a bottom-up way, so start at `3`

```
PushInt 3
```

```
square x = x * x
main = square (square 3)
```

The First Solution for Instantiation: G-Machine - 3

and then `square`

```
PushInt 3
PushGlobal square
```

```
square x = x * x
main = square (square 3)
```

The First Solution for Instantiation: G-Machine - 3

Now we can form an application node

```
square x = x * x
main = square (square 3)
```

```
PushInt 3
PushGlobal square
MakeApp
```

The First Solution for Instantiation: G-Machine - 3

Once we add another `square`

```
square x = x * x
main = square (square 3)
```

```
PushInt 3
PushGlobal square
MakeApp
PushGlobal square
```

The First Solution for Instantiation: G-Machine - 3

We can finish the main graph by constructing an application node

```
square x = x * x
main = square (square 3)
```

```
PushInt 3
PushGlobal square
MakeApp
PushGlobal square
MakeApp
```

The First Solution for Instantiation: G-Machine - 3

One more step here: we need to continue the graph reduction process

```
square x = x * x
main = square (square 3)
```

```
PushInt 3
PushGlobal square
MakeApp
PushGlobal square
MakeApp
Unwind
```

The First Solution for Instantiation: G-Machine - 4

Now let's compile `square` too

```
square x = x * x
main = square (square 3)
```

The First Solution for Instantiation: G-Machine - 4

We construct the arugment first

```
Push 0
```

```
square x = x * x
main = square (square 3)
```

The First Solution for Instantiation: G-Machine - 4

To construct its application node, we need to construct the function node

```
Push 0
Push 1
```

```
square x = x * x
main = square (square 3)
```

The First Solution for Instantiation: G-Machine - 4

```
square x = x * x
main = square (square 3)
```

```
Push 0

Push 1

PushGlobal *
```

The First Solution for Instantiation: G-Machine - 4

```
square x = x * x
main = square (square 3)
```

```
Push 0
Push 1
PushGlobal *
MakeApp
```

The First Solution for Instantiation: G-Machine - 4

and then the top-level application node

```
square x = x * x
main = square (square 3)
```

```
Push 0
Push 1
PushGlobal *
MakeApp
MakeApp
```

The First Solution for Instantiation: G-Machine - 4

Now, clean up the old root

```
square x = x * x
main = square (square 3)
```

```
Push 0
Push 1
PushGlobal *
MakeApp
MakeApp
Update 2
Pop 2
```

The First Solution for Instantiation: G-Machine - 4

Note that we `Update` the root here to share the work

```
square x = x * x
main = square (square 3)
```

```
Push 0
Push 1
PushGlobal *
MakeApp
MakeApp
Update 2
Pop 2
```

The First Solution for Instantiation: G-Machine - 4

Again, for the further evaluation, we need to add the `Unwind` instruction

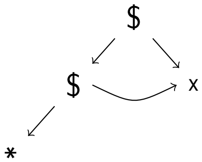
```
square x = x * x
main = square (square 3)
```

```
Push 0
Push 1
PushGlobal *
MakeApp
MakeApp
Update 2
Pop 2
Unwind
```

Problem Solved! Or... Did It? - 1

What does `Unwind` at the end of `square` do?

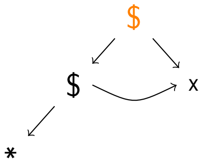
```
square x = x * x
main = square (square 3)
```



Problem Solved! Or... Did It? - 1

It first checks whether the root is a global/primitive value

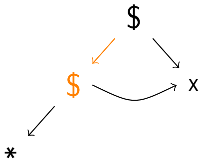
```
square x = x * x
main = square (square 3)
```



Problem Solved! Or... Did It? - 1

Otherwise, it steps into the function side

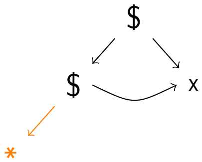
```
square x = x * x
main = square (square 3)
```



Problem Solved! Or... Did It? - 1

Until it reaches a global/primitive value

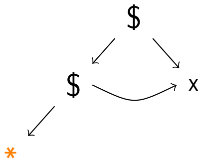
```
square x = x * x
main = square (square 3)
```



Problem Solved! Or... Did It? - 1

And jump to the code for the global

```
square x = x * x
main = square (square 3)
```



Problem Solved! Or... Did It? - 2

`Unwind` has linear time complexity to the length of a “spine” (the curried applications on a function)

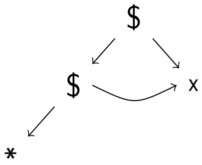
The Second Solution for Instantiation: TIM - 1

To avoid the problem of a spine, we will use “closures” to represent a call

The Second Solution for Instantiation: TIM - 1

To avoid the problem of a spine, we will use “closures” to represent a call

```
square x = x * x
```

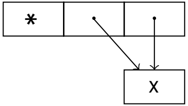


Not this

The Second Solution for Instantiation: TIM - 1

To avoid the problem of a spine, we will use “closures” to represent a call

```
square x = x * x
```



But this

The Second Solution for Instantiation: TIM - 2

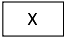
How can we build this?

```
square x = x * x
```

The Second Solution for Instantiation: TIM - 2

First, we take an argument

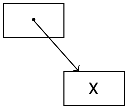
```
square x = x * x
```



The Second Solution for Instantiation: TIM - 2

Then, we put it into stack twice for the `*` call

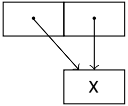
```
square x = x * x
```



The Second Solution for Instantiation: TIM - 2

Then, we put it into stack twice for the $\ast$ call

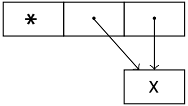
```
square x = x * x
```



The Second Solution for Instantiation: TIM - 2

Now we invoke $*$

`square x = x * x`



The Second Solution for Instantiation: TIM - 3

We can translate this into a code form

```
square x = x * x
```

The Second Solution for Instantiation: TIM - 3

First, we take an argument

Take 1

```
square x = x * x
```

The Second Solution for Instantiation: TIM - 3

Then, we put it into stack twice for the $*$ call

```
square x = x * x
```

```
Take 1
Push (Arg 0)
```

The Second Solution for Instantiation: TIM - 3

Then, we put it into stack twice for the `*` call

```
square x = x * x
```

```
Take 1
Push (Arg 0)
Push (Arg 0)
```

The Second Solution for Instantiation: TIM - 3

Now we invoke $*$

```
square x = x * x
```

```
Take 1
Push (Arg 0)
Push (Arg 0)
Enter (Label *)
```

Now it is really done, right? ...Right?

Note that we do not have anything corresponds to `Update` here.

In fact, only with these 3 instructions, we lose sharing of evaluations!

Now it is really done, right? ...Right?

In fact, `Update` is one of the key issue of TIM.
It is quite costly to have a correct sharing with
TIM due to its machine-level representation details

Now it is really done, right? ...Right?

Can we combine G-machine's simple memory structure (only pointers) and TIM's "closure"-based approach?

Let there be GHC

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- G. L. Burn, S. L. P. Jones, and J. D. Robson. “The spineless G-machine”. LFP’88

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- S. L. P. Jones. “Implementing lazy functional languages on stock hardware: the Spineless Tagless G-machine”. JFP’92

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- L. Maurer, P. Downen, Z. M. Ariola, and S. L. P. Jones. “Compiling without continuations”. PLDI’17

In summary

- In lazy evaluation, graph reduction is the key to handle sharing
- G-machine solves instantiation inefficiency but is still inefficient in its `Unwind`
- TIM removes some inefficiency of G-machine, but it also adds some for `Update`

- Their combinations can be more efficient...
  For that, see spineless tagless G-machine and "compiling without continuation"